Name:

SA402 · Dynamic and Stochastic Models Fall 2022 · Uhan

Quiz 7 – 11/4/2022

Instructions. You have 20 minutes to complete this quiz. You may use your plebe-issue calculator. You may use your own course materials (e.g., notes, homework, website). No collaboration allowed.

Show all your work. To receive full credit, your solutions must be completely correct, sufficiently justified, and easy to follow.

Problem	Weight	Score
1	1	
2	1	
3	1	
4	1	
Total		/ 40

For Problems 1-3, consider the Markov chain defined by the following one-step transition matrix:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

There are two irreducible sets of states: $\{2, 3\}$ and $\{5\}$.

Problem 1. Is state 3 transient or recurrent? Briefly explain.

Problem 2. Suppose the Markov chain has reached state 3. What is the steady-state probability of being in state 2?

Here is the one-step transition matrix from the previous page, for your convenience:

$$\mathbf{P} = \begin{bmatrix} 0.1 & 0.2 & 0.4 & 0.1 & 0.2 \\ 0 & 0.6 & 0.4 & 0 & 0 \\ 0 & 0.9 & 0.1 & 0 & 0 \\ 0.5 & 0.2 & 0.2 & 0 & 0.1 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 3. Suppose the Markov chain starts in state 4. What is the probability that the Markov chain is absorbed into state 5?

Problem 4. Consider a model of an elevator's movement from floor to floor in a high-rise building, in which the state of the system is defined as the floor on which the elevator is currently stopped, and the time index is defined to be the number of stops. Describe what assumptions need to be made in order for the Markov property to hold. (You do not need to discuss whether these assumptions are realistic.)